Aging in a system composed of Kuramoto Oscillators

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What is Synchrony?

Spontaneous order
Order in space
Order in time

Synchrony
Synchrony in Nature
Kuramoto Model

\[ \frac{d \phi_i}{dt} = \omega_i + \sum_{j}^{N} K_{ij} \sin(\phi_j - \phi_i) \]

\( \omega_i \rightarrow \text{Natural Frequency} \)

\( K_{ij} \rightarrow \text{Coupling Matrix} \)

Why Kuramoto Model?

Aging

Preparing the system

$t_{\text{waiting}}$

Perturbing the system

Measuring the response to perturbation

$t_w$ dependent response

Main characteristics of aging

- Slow relaxation
- Breaking of time translation invariance
- Dynamical scaling

Aging is observed in:

- Structural glasses
- Spin glasses
- Magnetic systems
- Colloids
\[
\frac{\partial \phi_i}{\partial t} = \omega + \frac{\kappa}{N_i} \sum_{j}^{N} \sin(\phi_j - \phi_i) + F(\vec{\phi})
\]

Homogeneous frequencies

\(\kappa = -|\kappa| \) repulsive coupling

\(N_i = 6 \) Hexagonal lattice

External Field

\(c \sin(\phi_i)\)

Oscillatory

Constant

\(d\)

Frustrated bonds
\[ C(t, t_w) = \frac{\langle \tilde{\phi}(t)\tilde{\phi}(t_w) \rangle - \langle \tilde{\phi}(t) \rangle \langle \tilde{\phi}(t_w) \rangle}{\sigma_t \sigma_{t_w}} \]

\[ c(t, t_w) = C(t, t_w) - C_{\text{plateau}} \]
\[ F(\varphi_i) = (0.1) \sin(\varphi_i) \]

\[ F(\varphi_i) = \sin(\varphi_i) \]
F(\phi_i) = 0.1

F(\phi_i) = 1
$c(t, t_w) \propto t_w^b e^{a(t-t_w)^\alpha}$

\begin{align*}
&\begin{align*}
  a &= -6.1 \times 10^{-4} \\
  \alpha &= 1.024 \\
  b &= 0.97
\end{align*} \\
&\begin{align*}
  a &= -1.13 \times 10^{-4} \\
  \alpha &= 1.162 \\
  b &= 0.837
\end{align*}
\end{align*}

$F(\phi_i) = (0.1)\sin(\phi_i)$

$F(\phi_i) = \sin(\phi_i)$
\[ c(t, t_w) \propto t_w^b e^{a(t-t_w)^\alpha} \]

\[ a = -2.4 \times 10^{-4} \]
\[ \alpha = 1.07 \]
\[ b = 1.219 \]
\[ F(\varphi_i) = 0.1 \]

\[ a = -2 \times 10^{-2} \]
\[ \alpha = 9.5 \times 10^{-1} \]
\[ b = 1.166 \]
\[ F(\varphi_i) = 1 \]
Summary

\[ \frac{\partial \varphi_i}{\partial t} = \omega + \frac{\kappa}{N_i} \sum_{j}^{N} \sin(\varphi_j - \varphi_i) + F(\varphi) \]

\[ \begin{cases} c \sin(\varphi_i) \\ d \end{cases} \]

Novel scaling behavior

\[ c(t, t_w) \propto t_w^b e^{a(t-t_w)^\alpha} \]

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<th>$c$</th>
<th>$a$</th>
<th>$\alpha$</th>
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Thank You