Non-equilibrium relaxation of driven topological defects

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Topological defects

'Mesoscopic' continuum description of condensed matter system: $n$-component order parameter field, assume $O(n)$ symmetry

→ **topological defect**: singularity, with integer ‘winding number’ $k$

- $n = 1$: scalar Ising field, discrete $Z_2$ up/down symmetry
  → **domain wall**

- $n = 2$: xy model, continuous planar rotational symmetry
  → **vortices, anti-vortices** $k = \pm 1$;
  also for complex order parameter with $U(1)$ gauge symmetry

- $n = 3$: Heisenberg model, continuous rotational symmetry
  → **hedgehogs, ‘skyrmions’**
Physical ‘aging’

Prepare (non-linear, stochastic) dynamical system in initial state ‘far away’ from long-time asymptotic steady-state configuration:

- **steady state stationary**: $\langle S(t) \rangle = \langle S(0) \rangle$ time-independent, two-point correlations: $C(s, t) = \langle S(s)S(t) \rangle_c = C(t - s)$
- **initial state breaks time-translation invariance**
- ‘slow’ dynamics: transient time window becomes accessible
dynamic scaling:

$$C(x, t, s)_c = \langle S(0, s)S(x, t) \rangle_c = s^{-b} \hat{C}\left(\frac{x}{L(t)}, \frac{L(s)}{L(t)}\right)$$

**characteristic length**: $L(t) \sim t^{1/z} \rightarrow$ **dynamic exponent** $z$

**autocorrelation** function ($x = 0$) in ‘aging’ scaling regime:

$s \ll t : C(0, t, s)_c = s^{-b} \hat{C}(t/s) \sim s^{-b} [L(s)/L(t)]^\lambda \sim t^{-\lambda/z}$

→ non-trivial information about **fluctuations, correlations**

- scaling exponents $b$, $z$, $\lambda$, and scaling functions **universal**?
- or rather **characteristic** of specific material properties?
Type-II superconductors, magnetic flux lines

London penetration depth/coherence length $\lambda/\xi > \sqrt{2}$:

- **negative** interfacial free energy $\rightarrow$ mixed phase (normal/SC)
- $H_{c1} = 4\pi\epsilon/\Phi_0 < H < H_{c2} = \Phi_0/2\pi\xi^2$: $n = B/\Phi_0$ flux lines
- **elastic line tension**: $\epsilon = (\Phi_0/4\pi\lambda)^2 \ln(\lambda/\xi)$, $\Phi_0 = hc/2e$
- **vortex repulsion**: $V_{\text{int}}(r) = 2\epsilon_0 K_0(r/\lambda) \rightarrow$ Abrikosov lattice
- **thermal wandering**: flux liquid, vortex motion $\rightarrow$ dissipation effective flux pinning required $\rightarrow$ material defects, smooth $V_D$
Theoretical description

London limit $\lambda \gg \xi$: vortices ↔ trajectories $r_j(z) \rightarrow$ elastic lines
employ grand-canonical ensemble, with $\mu = (H - H_{c1})\Phi_0/4\pi$:

$$H_N = \int_0^L dz \sum_{j=1}^N \left[ \frac{\epsilon}{2} \left| \frac{dr_j(z)}{dz} \right|^2 + V_D(r_j(z)) + \frac{1}{2} \sum_{i \neq j} V_{int}(r_{ij}(z)) \right]$$

$$Z_{gr}(T, H) = \sum_{N=0}^{\infty} \frac{e^{\mu NL/k_B T}}{N!} \int D[r_j(z)] e^{-H_N[r_j(z)]/k_B T}$$

Langevin Molecular Dynamics:

- discretize into layers $z$ perpendicular to magnetic field $B$
- assume overdamped dynamics, Bardeen–Stephen viscosity $\eta$
- uncorrelated Gaussian thermal noise $f_{j,z}$
- average over stochastic noise ‘histories’:

$$\eta \frac{\partial r_{j,z}(t)}{\partial t} = - \frac{\delta H[r_{j,z}(t)]}{\delta r_{j,z}(t)} + f_{j,z}(t)$$

$$\langle f_{i,z}(t) \cdot f_{j,z'}(t') \rangle = 4\eta k_B T \delta_{ij} \delta_{zz'} \delta(t - t')$$

parameters $\sim$ YBCO; 16 vortices, 1116 \ldots 1710 pins per layer
Temperature quenches: point pins vs. columnar defects

- Initialize system: *randomly placed straight* vortex lines
- Evolve and relax for 100,000 time steps
- Instantaneously raise temperature

Measure gyration radius \( r_g(t) = \sqrt{\langle [r_{j,z}(t) - \langle r_j(t) \rangle]^2 \rangle} \)

Isolated point pins

- Exponential decay; no aging signatures
  - Point defects: \( \tau = 3.4 \cdot 10^4 \): enhanced thermal wandering
  - Linear pins: \( \tau_1 = 1.6 \cdot 10^3 \), \( \tau_2 = 5.7 \cdot 10^4 \): relax double kinks
Magnetic field / vortex density quenches

evaluate two-time `height' autocorrelation function:
\[ C(t, s) = \langle (r_j(z(t)) - \langle r_j(t) \rangle) (r_j(z(s)) - \langle r_j(s) \rangle) \rangle \]

- initialize and relax system for duration \( r = 100,000 \) time steps
- suddenly change magnetic field \( \rightarrow \text{add or remove} \) flux lines
- new vortices: \( s, t \); original vortices: \( \sigma = r + s, \Gamma = r + t \)

(a) **fixed flux density**: system relaxed
(b) **field down-quench**: 21 \( \rightarrow \) 16 lines aging

(c) **field up-quench**: 16 \( \rightarrow \) 21 lines aging
(d) only 5 **added lines**: non-monotonic dynamics
Current quenches: correlated driven initial state

compute normalized two-time ‘height’ autocorrelation function:

\[
C(t, s) = \frac{\langle (r_{j,z}(t) - \langle r_j(t) \rangle)(r_{j,z}(s) - \langle r_j(s) \rangle) \rangle}{\langle (r_{j,z}(s) - \langle r_j(s) \rangle)^2 \rangle}
\]

- stay within moving regime: exponential relaxation, no aging
- quench from moving to pinned glassy phase: clear aging; scaling exponents: \( b \approx 0.005, \lambda_C/z \approx 0.011 \)

see Harsh Chaturvedi, contributed talk, session M 2.1
Relaxation dynamics of skyrmions in magnetic films

**magnetic skyrmions:**
- two-dimensional *chiral* magnet
- no inversion symmetry
- weak perpendicular magnetic field $\vec{H}$

**coarse-grained** energy: ferromagnetic exchange, helical interaction

$$H = \int d^2r \left[ J (\nabla \hat{\mathbf{n}})^2 + D \hat{\mathbf{n}} \cdot \vec{\nabla} \times \hat{\mathbf{n}} - \vec{H} \cdot \hat{\mathbf{n}} \right]$$

effective particle model: $\eta \mathbf{v}_i = \mathbf{F}_{i}^{M} + \mathbf{F}_{i}^{s} + \mathbf{f}_i$
- Magnus Force: $\mathbf{F}_{i}^{M} = \beta \hat{\mathbf{z}} \times \mathbf{v}_i$
- skyrmion repulsion: $\mathbf{F}_{i}^{s} = F_0 \sum_{j \neq i} K_1(r_{ij}) \hat{\mathbf{r}}_{ij}$
- thermal white noise:
  $$\langle \mathbf{f}_i \rangle = 0, \quad \langle \mathbf{f}_i(t) \cdot \mathbf{f}_j(t') \rangle = \sigma \delta_{ij} \delta(t - t'), \quad \sigma = 4\eta k_B T$$

two-time density autocorrelation function (149 skyrmions):

$$n_i = 0, 1: \quad C(t, s) = \langle n_i(s) n_i(t) \rangle$$

(a) $t = s$  
(b) $t > s$  
(c) $t >> s$
Skyrmion relaxation kinetics following temperature quench

Mean nearest-neighbor distance and scaled density autocorrelation:

- no Magnus force ($\beta = 0$):
- with Magnus force, $\beta/\eta = 1$:

\[ L(t) \]

\[ t \]

\[ \sigma \]

\[ b \]

\[ 0.0 \quad 0.1 \quad 0.2 \quad 0.5 \]

\[ -0.2 \quad -0.1 \quad 0.0 \quad 0.33 \]

→ Aging scaling depends on temperature, Magnus force strength

See Bart Brown, contributed talk, session G 2.4
Summary and conclusions

- 3D elastic line model for disordered type-II superconductors
- utilized Langevin molecular dynamics
- studied external parameter quenches: sudden changes of temperature, magnetic field, driving current
- slow (algebraic, logarithmic) relaxation → physical aging
- (approximate) aging scaling exponents non-universal: depend on physical parameters, disorder, and initial states
- 2D particle model for skyrmions in chiral magnetic films
- temperature quenches: aging modified by Magnus force
- non-equilibrium relaxation: tool for materials characterization