Understanding Hadronization from SIDIS processes.

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Motivation

Phenomenology of SIDIS and TMDs

SIDIS and the current fragmentation region: physical picture

Power counting and kinematics of the current region

Final remarks
Motivation

We would like to learn about Hadron Structure. More generally about the basic mechanisms for Confinement and Hadronization.

Theoretical tool: QCD, separation of hard and soft effects, factorization theorems.
Motivation

Semi-inclusive deep inelastic scattering
Semi-inclusive deep inelastic scattering

A case of great interest, transverse momentum dependent distributions and fragmentation functions (TMDs)
Motivation

Semi-inclusive deep inelastic scattering

TMDs are Non-perturbative functions, need to determine them from data. Many complications in phenomenological applications.
Phenomenology of SIDIS and TMDs

Two stages (My view)
Warning: not a comprehensive review.

Simple models:
gather as much intel as possible
easy to implement, generally leads to interpretations of limited validity
Phenomenology of SIDIS and TMDs

Two stages (My view)

Simple models: gather as much intel as possible
easy to implement, generally leads to interpretations of limited validity

Full QCD picture: perturbative corrections, evolution equations ...
Ultimate goal, great predictive power. Many obstacles yet remain.
Phenomenology of SIDIS and TMDs


gaussian model

\[ f_q(x, k_{\perp}) = f_q(x) \frac{1}{\pi\langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2/\langle k_{\perp}^2 \rangle} \]

\[ D^h_q(z, p_{\perp}) = D^h_q(z) \frac{1}{\pi\langle p_{\perp}^2 \rangle} e^{-p_{\perp}^2/\langle p_{\perp}^2 \rangle} \]

\[ z_h = \frac{\text{Ph} \cdot P}{q \cdot P} \]

\[ P_T \text{ observed hadron} \]

stage I

Anselmino, Boglione, et.al. (2005)
DOI: 10.1103/PhysRevD.71.074006

[Graphs and images related to the Phenomenology of SIDIS and TMDs]
Phenomenology of SIDIS and TMDs

stage I

Anselmino, Boglione, et.al. (2005)
DOI: 10.1103/PhysRevD.71.074006

\[ z_h = \frac{\text{Ph} \cdot \vec{P}}{q \cdot \vec{P}} \]

\[ \vec{P}_T \text{ observed hadron} \]

Data integrated over other kinematical variables:

\[ x_B, Q^2 = -q^2 \]
Phenomenology of SIDIS and TMDs

Multidimensional data

\[ z_h = \frac{P_h \cdot P}{q \cdot P} \]

PT observed hadron

\( x_B, Q^2 \)

Gaussian model

Describes well low-\( z_h \) and low \( P_T \)

\[ f_q(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle} \]

\[ D_q^h(z, p_\perp) = D_q^h(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-p_\perp^2 / \langle p_\perp^2 \rangle} \]
Phenomenology of SIDIS and TMDs

Gaussian model

Describes well a subset of low-$z_h$ and low $P_T$

Observe mild $Q^2$ dependence in data.

Need to think about TMD evolution, stage 2

stage 1
Phenomenology of SIDIS and TMDs

$q_T = P_T / z_h$

$W + Y$ construction by Collins-Soper-Sterman

$W$ describes the TMD region

$Y$ pQCD corrections at larger $q_T = P_T / z_h$
Phenomenology of SIDIS and TMDs

$q_T = \frac{P_T}{z_h}$

$W + Y$ construction
Collins-Soper-Sterman

Coming from gluon radiation, Collinear Factorization

$q_T \ll Q$
TMD region

$q_T \sim Q$
Matching region

$q_T \gg Q$
pQCD

Friday, November 11, 16
Phenomenology of SIDIS and TMDs

Can we describe the entire range of $q_T = P_T/z_h$?

W + Y construction
Collins-Soper-Sterman

Coming from gluon radiation, Collinear Factorization
Phenomenology of SIDIS and TMDs

Works for SIDIS at high enough, $Q^2 > 10$ GeV$^2$ (integrated over $z_h$)

Nadolsky, Stup, Yuan
DOI: 10.1103/PhysRevD.64.059903

$q_T = P_T/z_h$

Coming from gluon radiation, Collinear Factorization

Collins-Soper-Sterman

$W + Y$ construction
Can’t describe available multi-dimensional SIDIS data, $Q^2 < 10$ GeV$^2$

Need to understand why this is so. The first place to look at is the kinematics of SIDIS

Investigate the range of validity of the TMD approach, the so called current fragmentation region
SIDIS and the current fragmentation region

We need a quantitative way to identify the current region.
SIDIS and the current fragmentation region

It should be noted ...


Phase space should be large enough to distinguish current/target regions

(“Berger criterion”)

SIDIS and the current fragmentation region: physical picture

In collaboration with:

- Mariaelena Boglione (Torino)
- John Collins (Penn State)
- Leonard Gamberg (Penn State Berks)
- Ted Rogers (ODU/JLab)
- Nobuo Sato (JLab)

Notice this is work in progress!!!
factorization theorems

Fragmentation Functions

Fracture Functions
these regions are assumed to be well separated in the **observed hadron rapidity**

\[ y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-} \]
$y_h \equiv \frac{1}{2} \log \frac{P^+}{P^-}$

However, this neglects the soft fragmentation region

(No factorization theorem for this region)
One may take this into account, at least when defining **kinematic limits** for current/target region.
current region
(fragmentation functions)
Power counting and kinematics of the current region
Factorization implies a power counting for the quark momenta

\[ k_i = \left( O(Q), O(m^2/Q), O(m) \right) \]

\[ k_f = \left( O(m^2/Q), O(Q), O(m) \right) \]

\[ P_h \cdot k_f = O(m^2) \]

\[ |k_i^2| = O(m^2) \]

\[ P_h \cdot k_i = O(Q^2) \]

\[ k_f^2 = M_f^2 = O(m^2) \]
Factorization implies a power counting for the quark momenta

\[ k_i = \left( O(Q), O(m^2/Q), O(m) \right) \]

\[ k_f = \left( O(m^2/Q), O(Q), O(m) \right) \]

\[ P_h \cdot k_f = O\left( m^2 \right) \]

\[ P_h \cdot k_i = O\left( Q^2 \right) \]

\[ |k_i^2| = O(m^2) \]

\[ k_f^2 = M_j^2 = O(m^2) \]
This quantity must remain small.

\[ R(y_h, z_h, x_{bj}, Q) \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

\[ P_h \cdot k_f = O(m^2) \]

\[ P_h \cdot k_i = O(Q^2) \]

\[ |k_i^2| = O(m^2) \]

\[ k_f^2 = M_j^2 = O(m^2) \]

**current region**

**hard scale**

**small masses**
This quantity must remain small.

\[ R(y_h, z_h, \lambda, Q) \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

\[ P_h \cdot k_f = O(m^2) \]

\[ P_h \cdot k_i = O(Q^2) \]

\[ |k_i^2| = O(m^2) \]

\[ k_f^2 = M_j^2 = O(m^2) \]

current region

hard scale (large)
It seems simple enough to set a criterion by imposing a cut in rapidity.

At these kinematics, even for \( z_h = 0.2 \)
\( R \) remains small in a sizeable range of rapidity.
\[ R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

Quark momenta should be estimated

\[ k_i = \left( O(Q), O(m^2/Q), O(m) \right) \]

\[ k_f = \left( O(m^2/Q), O(Q), O(m) \right) \]

Note the uncertainty in the quark rapidities are unimportant
The picture starts to change when looking at lower values of $Q^2$

Assumptions about quark momenta become more relevant
\[ R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

The picture starts to change when looking at lower values of \( Q^2 \).

Current region shrinks, low values of \( z_h \) lie almost entirely outside.
\[ R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

For very low values of \( Q^2 \), things get fuzzy

It’s hard to establish a criterion (thick bands)
Estimated quark rapidities are dangerously close.

Within this picture, the current and non-current regions strongly overlap.
These likely are signals of the breaking of the formalism.

\[ R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

\( P_h \cdot k_f = O(m^2) \)

\( P_h \cdot k_i = O(Q^2) \)

Can’t tell precisely how large it should be.
New formalism for low Q2 ??

\[ y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-} \]
In the mean time

\[ R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

One may incorporate these considerations into phenomenological analyses by looking at regions of small \( R \).
In the mean time

\[ R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

Note this implies also a dependence on \( P_{hT} \), the transverse momentum of the observed hadron.
In the mean time

\[ R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i} \]

Alternatively, imposing rapidity cuts

\[ k_f \]

\[ k_i \]
Final remarks

Important to always keep track of the range of applicability of the formalism of fragmentation functions (self-consistency)

Requiring $R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$ to be small, simple test for current region

Kinematical constraints involve both $P_{hT}$ and $z_h$

Within the available formalisms, fragmentation and fracture functions may overlap at low $Q^2$ (how low?)

At low values of $Q^2$ the notion of current region starts to fade. Extension of CSS formalism needed, to include some soft gluon effects.