An Investigation of Light Ion Production from Coalescence in Nuclear Collisions

Wouter C. de Wet, Lawrence W. Townsend, Charles M. Werneth, Ryan B. Norman, John W. Norbury, Tony C. Slaba, William P. Ford

November 9–12, 2016

a – University of Tennessee, Department of Nuclear Engineering, Knoxville, TN
b – NASA Langley Research Center, Hampton, VA
Introduction

• Space is filled with HZE radiation

• Light ions (\(^1\)H, \(^2\)H, \(^3\)H, \(^3\)He, \(^4\)He) and neutrons make large contributions to dose equivalent\(^[1]\)

• Light ion production cross-sections are important for dosimetry and calculating radiation environments

• Coalescence is a mechanism of light ion production

• Relativistic Abrasion–Ablation & De–Excitation FRaGmentation Model (RAADFRG)
Objectives

- Nuclear Collision Model Overview
- Coalescence Formalism & Mathematical Derivations
- Coalescence Radius Parameter, $P_0$, Sensitivity Study
Nuclear Collision Overview

Time $\approx 0$ sec

Abrasions
Nuclear Collision Overview

Time $\approx 10^{-22}$ sec

Excited Pre-Fragment with cross-section, $\sigma_i$

Distribution of Abraded Nucleons

Abrasion
Nuclear Collision Overview

Time ≈ $10^{-16}$ sec

Distribution of Abraded Nucleons

Ablation
Nuclear Collision Overview

Time $\approx 10^{-22}$ sec

Abrasion

Distribution of Abraded Nucleons
Nuclear Collision Overview

Time $\approx 10^{-22}$ sec

Coalescence

Distribution of Abraded Nucleons

Coalesced Light Ions
Coalescence Formalism

For a coalesced particle with $Z$ protons, $N$ neutrons, and mass $A$, the differential multiplicity momentum distribution, $\frac{d^3N^i(Z,N)}{dp^3(Z,N)}$, is given by the equation below. Here, multiplicity is defined as the number of particles of species $Z, N$ formed by coalescence in abrasion reaction channel $i$.

$$\frac{d^3N^i(Z,N)}{dp^3(Z,N)} = \frac{\bar{m}^i_{(Z,N)}}{N!Z!} \left( \frac{4\pi}{3} P_0^{(Z,N)^3} \gamma \right)^{A-1} \left[ \frac{d^3N^i(0,1)}{dp^3} \right]^N \left[ \frac{d^3N^i(1,0)}{dp^3} \right]^Z$$

- $\frac{d^3N^i(0,1)}{dp^3}$ and $\frac{d^3N^i(1,0)}{dp^3}$ respectively represent the differential multiplicity distributions for abraded neutrons and protons in abrasion reaction channel $i$.
- $\bar{m}^i_{(Z,N)}$ is defined as the average multiplicity of the coalesced particle of species $Z, N$ in abrasion reaction channel $i$.
- $\gamma$ is the standard relativistic coefficient of the coalesced particle. Here, $\gamma \approx 1$.
- The parameter $P_0^{(Z,N)}$ is termed the coalescence radius and signifies the radius, in momentum space [MeV/c], of the coalescence volume.
Coalescence Formalism

Assuming the abraded proton and neutron momentum distributions are of the same shape, they may be related as follows:

\[
\frac{d^3 N^i(0,1)}{dp^3} = \left( \frac{\bar{m}^i_{(0,1)}}{\bar{m}^i_{(1,0)}} \right) \frac{d^3 N^i(1,0)}{dp^3}
\]

- Abraded proton and neutron multiplicities are represented by \( \bar{m}^i_{(1,0)} \) and \( \bar{m}^i_{(0,1)} \), respectively.

The cross-section and multiplicity may be related using:

\[
\frac{d^3 N^i_k}{dp^3_k} = \frac{1}{\sigma_{\text{abr}}^i} \frac{d^3 \sigma^i_k}{dp^3_k}
\]

- \( \sigma_{\text{abr}}^i \) represents the cross-section of abrasion channel \( i \).
- \( \frac{d^3 N^i_k}{dp^3} \) is the differential multiplicity distribution of particle \( k \) produced in abrasion channel \( i \).
- \( \frac{d^3 \sigma^i_k}{dp^3} \) is the differential yield cross-section distribution.
Coalescence Formalism

Assuming a Gaussian momentum distribution for abraded nucleons, the differential cross-section for forming composite nucleus $Z, N$ is given by:

$$\frac{d^3 \sigma^i(Z, N)}{dp^3} = \frac{\bar{m}^i_{(Z,N)}}{N! Z!} \left( \frac{4\pi}{3\sigma^i_{abr}} p_0^{(Z,N)^3} \right)^{A-1} \left( \frac{\bar{m}^i_{(0,1)}}{\bar{m}^i_{(1,0)}} \right)^N \left[ \frac{\sigma^i(1,0)}{(2\pi\Delta^2)^{3/2}} \right]^A e^{(-\frac{Ap^2}{2\Delta^2})}$$

Where $\Delta$ is a parameterization of the distribution variance given by:

$$\Delta = \begin{cases} 
232 - 2.3 \cdot A_{parent} & A_{parent} \leq 56 \\
104 - 0.043 \cdot A_{parent} & A_{parent} > 56 
\end{cases}$$

Integrating over momentum gives:

$$\sigma^i(Z, N) = \frac{\bar{m}^i_{(Z,N)}}{N! Z!} \left( \frac{4\pi}{3\sigma^i_{abr}} p_0^{(Z,N)^3} \right)^{A-1} \left( \frac{\bar{m}^i_{(0,1)}}{\bar{m}^i_{(1,0)}} \right)^N \frac{A^{3/2}}{(2\pi\Delta^2)^{3/2(A-1)}} \sigma^i(1,0)^A$$
Coalescence Formalism

- Final form:

\[ \sigma^i(Z, N) = \frac{\sigma_{\text{abr}}^i}{N!Z!} \left( \frac{4\pi}{3} p_0^{(Z,N)} \right)^3 \frac{A^{-1}(\bar{m}_i^{(0,1)})^N(\bar{m}_i^{(1,0)})^{1-N} A^Z}{(2\pi \Delta^2)^\frac{3}{2}(A-1)} \]

- As implemented:

\[ \sigma_{\text{coal}}(Z, N) = \sum_i \sigma^i(Z, N) \]
## Coalescence of Abraded Nucleons

### NUCFRG3
- Coalescence is calculated for the entire abrasion reaction as a single event.
- \( P_0 \) parameter is expected to account for many physical details.
- Abraded nucleon multiplicity distributions come from the projectile nucleus.

### RAADFRG
- Abrasion reaction is the sum of individual abrasion reaction channels.
- Coalescence contributions from each abrasion channel are calculated individually.
- Abraded nucleon multiplicity is explicitly known for each abrasion channel.
- Does not allow for non-physical coalescence events.
## Coalescence Radius

<table>
<thead>
<tr>
<th>Author</th>
<th>Deuteron</th>
<th>Triton</th>
<th>Helium-3</th>
<th>Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nagamiya et al. [2]</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>Gutbrot et al. [3]</td>
<td>125.0</td>
<td>129.0</td>
<td>129.0</td>
<td>140.0</td>
</tr>
<tr>
<td>Awes et al. [4]*</td>
<td>170.0</td>
<td>215.0</td>
<td>240.0</td>
<td>270.0</td>
</tr>
<tr>
<td>RAADFRG</td>
<td>99.0</td>
<td>144.0</td>
<td>144.0</td>
<td>222.0</td>
</tr>
</tbody>
</table>

These values correspond to the coalescence radii used to perform the calculations on the following slides. Note that NUCFRG3 uses the same values listed here under Nagamiya et al.

*Recommended $P_0$ values are scaled x0.6
$^{12}\text{C} + ^{27}\text{Al} \rightarrow 1050 \text{ MeV/n}$

Light Ion Coalescence Cross-Sections

- Olson & Lindstrom [5,6]
- Gutbrod et al. – Coalesced
- Awes et al.* – Coalesced
- NUCFRG3/Nagamiya et al. – Coalesced
- RAADFRG – Coalesced

*Recommended P_0 values are scaled x0.6
$^{12}\text{C} + ^{27}\text{Al} \rightarrow 1050 \text{ MeV/n}$

Light Ion Production Cross-Sections

*Recommended $P_0$ values are scaled x0.6
\[ ^{12}\text{C} + ^{27}\text{Al} \rightarrow 1050 \text{ MeV/n} \]

**Light Ion Production Cross-Sections**

- Olson & Lindstrom [5,6]
- NUCFRG3/Nagamiya et al. – Coalesced
- NUCFRG3/Nagamiya et al. – Final
- RAADFRG – Coalesced
- RAADFRG – Final

The ‘Final’ cross-section is the sum of contributions from coalescence, electromagnetic dissociation, ablation, and residual pre-fragments.
$^{12}\text{C} + ^{12}\text{C} - 1050 \text{ MeV/n}$

Light Ion Production Cross-Sections

- Olson & Lindstrom [5,6]
- NUCFRG3/Nagamiya et al. - Coalesced
- NUCFRG3/Nagamiya et al. - Final
- RAADFRG - Coalesced
- RAADFRG - Final

Fragment Charge, Z

Cross-Section (mb)
$^{12}\text{O} + ^{12}\text{C} - 2100$ MeV/n

Light Ion Production Cross-Sections

Olson & Lindstrom [5,6]
NUCFRG3/Nagamiya et al. – Coalesced
NUCFRG3/Nagamiya et al. – Final
RAADFRG – Coalesced
RAADFRG – Final
\( ^{12}\text{O} + ^{27}\text{Al} \rightarrow 100 \text{ MeV/n} \)

**Light Ion Production Cross-Sections**

*Experimental data is normalized to fit theoretical reaction cross section.*
Conclusion

• The NUCFRG3 coalescence model has been updated for implementation into RAADFRG

• This coalescence model yields reasonable results for the reactions shown

• Coalescence cross-sections are very sensitive to the coalescence radius parameter

• New $P_0$ values agree well with experimental data for various projectile–target–energy combinations
Reference


Questions?