Relaxation Dynamics of Interacting Skyrmions in Thin Films

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Magnetic Skyrmions are particle-like spin textures which were recently shown to exist in chiral magnets without inversion symmetry in the presence of a weak magnetic field of $\sim 100 \text{mT}$ (applied $\perp$ to the plane in thin films).

- Topological winding number:
  \[ Q = \frac{1}{4\pi} \int dxdy \vec{n} \cdot (\partial_x \vec{n} \times \partial_y \vec{n}) \]
- Can be moved by very low current densities

Coarse-Grained Thin Film Hamiltonian

The local magnetic moment is described by the unit vector \( \hat{n}(\vec{r}, t) \).

\[
\mathcal{H} = d \int d\vec{r}^2 \left[ \frac{J_{ex}}{2} (\nabla \hat{n})^2 + D\hat{n} \cdot \nabla \times \hat{n} - \vec{H}_a \cdot \hat{n} \right]
\]

- Film thickness, \( d \ll \) skyrmion length scale
- Ferromagnetic exchange interaction favors spin alignment
- Antisymmetric exchange interaction favors helical ordering
- Weak (~ 100 mT) magnetic field applied \( \perp \) to thin film

Mühlbauer, Binz, Jonietz, Pfleiderer, Rosch, Neubauer, Georgii, Böni. Science 323, 915 (2009)
Particle Based Model

\[ \alpha \vec{v}_i = \vec{F}_i^M + \vec{F}_i^{ss} + \vec{F}_i^T \]

- Magnus force acting on the \( i_{th} \) skyrmion:
  \[ \vec{F}_i^M = \beta \hat{z} \times \vec{v}_i \]

- The repulsive force between skyrmions:
  \[ \vec{F}_i^{ss} = \sum_{j \neq i} F_0^{ss} K_1(r_{ij}) \hat{r}_{ij} \]

- Assume a Gaussian noise term:
  \[ \langle F^T_\mu \rangle = 0, \langle F^T_{i,\mu}(t)F^T_{j,\nu}(t') \rangle = \sigma \delta(t - t') \delta_{ij} \delta_{\mu\nu} \]


Simulation Details

- Skyrmions interact on a periodic domain with an aspect ratio of $2/\sqrt{3}$.
- The Langevin equation is solved using a 4th order RK method.
- At $t = 0$ the simulation is quenched to a finite noise.
- The number of skyrmions in each simulation is 149.

After a temperature quench at $t = 0$, (a), skyrmions relax into a triangular lattice configuration to minimize interactions, (b, c).
Interacting skyrmions in the zero noise limit with $\beta/\alpha = 0$. The particles relax algebraically in time into a triangular lattice.
The non-zero Magnus force causes rotations as the skyrmions relax into a triangular lattice configuration.
Two-Time Density Correlation Function

Consider the occupation number $n_i(t)$ for the $i_{th}$ skyrmion.

\[
n_i(t) = \begin{cases} 
0 & : \text{$i_{th}$ particle is outside of the region at time $t$} \\
1 & : \text{$i_{th}$ particle occupies the region at time $t$}
\end{cases}
\]

\[
C(t, s) = \langle n_i(t)n_i(s) \rangle
\]

- Circles are drawn about each skyrmion at the waiting time $s$.

(a) $t = s$

(b) $t > s$

(c) $t \gg s$

The skyrmions move out of their circles as they relax into the triangular lattice configuration so that in the example above, all $n_i = 1$ in (a) and 0 in (c).

Results: Without the Magnus Force, $\beta/\alpha = 0$

We measure the average nearest neighbour distance as a function of time, $L(t)$, and $C(t, s)$ for various waiting times as well as different values of $\sigma$. We find that $C(t, s)$ has a non-universal scaling exponent, $b$, that is a function of $\sigma$.

- As the skyrmions relax, the length scale evolves in time as a power-law before the skyrmions are captured.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>-0.2</td>
<td>-0.1</td>
<td>0.0</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Results: With the Magnus Force, $\beta/\alpha = 1$

We turn on the Magnus force and likewise measure $L(t)$ and $C(t, s)$. The Magnus force drives nearest neighbours closer on average which further lowers $L(t)$ for large $\sigma$, but helps the system relax into the triangular lattice for low $\sigma$.

- The skyrmions become captured very quickly in the $\sigma = 0.5$ case, obscuring the power law regime.

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Conclusions

- Relaxation into the triangular lattice is not described by a universal scaling law. The scaling exponent depends on both $\sigma$ and $\beta/\alpha$.
- After the formation of the lattice, the Magnus force drives nearest neighbours closer on average.
- The Magnus force causes rotations during the formation of the triangular lattice at low $\sigma$ which has the effect of increasing the magnitude of $b$ for low values of $\sigma$. 