Parallel Critical Fields in Niobium: Comparison to Theory

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- Prof. C.B. Eom (Univ of Wisconsin-Madison) for XRD work
Introduction

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- Can we compare to the theoretical predictions?
From Tinkham, if the film is thin enough ($d_F < 1.84\xi(T)$),

$$H_{c2\parallel}(T) = \frac{\phi_0}{\pi\xi(T)}\frac{\sqrt{3}}{d_F}$$

(if film is dirty!)
Theory of Parallel Critical Field for Thin Superconducting Films

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- Enhancement in $T_c$ for $d_F < d_c$ where $d_c^2 = 7.2\gamma(\lambda_{tr})\ell^2$. 

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- Enhancement in \(T_c\) for \(d_F < d_c\) where \(d_c^2 = 7.2\gamma(\lambda_{tr})\ell^2\).
- \(\ell\) is the elastic mean free path, \(\lambda_{tr} = \xi/\ell\), \(\xi = \frac{\hbar v_F}{2\pi k_B T_c}\) is a coherence length, and \(\gamma(x)\) is given by

\[
\gamma(x) = \frac{x^2 \sum_{n=0}^{\infty} (2n+1)^{-2}(2n+1+x)^{-3}}{\sum_{n=0}^{\infty} (2n+1)^{-2}(2n+1+x)^{-1}}.
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Later work by Hara and Nagai, J. Phys. Soc. Japan, 63, 2331 (1994): Parallel critical field can be expanded as

\[ t_C(\lambda) = 1 + C_2 \lambda^2 + \cdots \]

where \( t_C = T_c(B)/T_c(0) \), \( \lambda = (eB/\hbar)\xi^2 \) and \( \xi = (\hbar v_F/(2\pi k_B T_c(0))) \).

From their paper, \( C_2 \) can be calculated as

\[ C_2 = \sum_{n=0}^{\infty} \left( \frac{8}{15} \tilde{\epsilon}_3 \epsilon_2^n - \frac{2}{9} \tilde{\epsilon}_2 \epsilon_2^n - \frac{8}{27} \tilde{\epsilon}_4 \epsilon_2^n - \frac{2}{3} \tilde{\epsilon}_1 \epsilon_2^n \right) \]
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$$t_C = \frac{T_c(B)}{T_{c0}}, \quad \lambda = \frac{eB}{\hbar} \xi^2$$

and

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where \( I_n = \int_0^{\pi/2} \sin^3(\theta) \cos^3(\theta) \tanh \left( \frac{\tilde{\epsilon}_n d}{2 \cos(\theta)} \right) d\theta, \)

We will have to account for strong coupling, which the theory does not.

\( \eta Bc^2 \approx 1.06 \) for our niobium.

Showed \( C_2 < 0 \) always, while Kogan claimed for clean enough and thin enough, \( C_2 > 0 \) near \( t \approx 1. \)
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Nb films grown by magnetron sputtering onto (100) oriented Si or A-plane/C-plane Sapphire substrates

Substrate Stage at Room Temperature or 300°C

Base Pressure <4 x 10^{-8} Torr with LN$_2$ cooled surface

Deposition Rates ≈ 8 nm/minute

Film thicknesses from 15 to 100 nm

Argon sputter pressure varied from 0.5-5 mTorr
Best Result: XRD along film normal

S (11-20)
2θ : 37.786°
Rocking FWHM : 0.008°

Thickness
~25nm from fringes

Nb (110)
2θ : 38.6342°

Nb Rocking FWHM: 0.02°
Best Result: XRD pole plot

Schematic pole-figure

Parallel Critical Fields in Niobium: Comparison to Theory
Measurement Method for Critical Fields

- Critical Fields determined by Van der Pauw resistive transition.
- Sample current at 1 mA, $B_{c2||}$ determined by midpoint.
- CERNOX thermometer, OFHC platform, Cryocooler
- Multiple trials to ensure consistency.
- Maximum field is 707 mT
Transport for 25 nm Nb film

Resistance per square vs Temperature

Parallel Critical Field

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2D behavior for 48 nm Nb film

\[ T = 8.00 \text{ K} \quad (t=0.967) \]

**Tinkham 2D formula**

- **Graph**
  - **Y-axis**: \( B_{c2}(t) \) (mT)
  - **X-axis**: Angle between \( B \) and film normal (°)

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Parallel Critical Fields in Niobium: Comparison to Theory
Comparison to Theory for 25 nm Nb film ($d_c \approx 30$ nm)
Comparison for $|C_2|$ for $d_F/\xi \approx 0.6$
Comparison for all films

![Graph showing experimental vs. theoretical values of $|C_2|$ for all films. The graph compares $|C_2|$ experimental values against $|C_2|$ theoretical values, with data points indicating a close alignment with the theoretical line.](image-url)
Comparison for smaller values
Conclusions

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- Question on strong coupling correction: $\eta_{Bc2}$ or $\eta_{Bc2}^2$?
- Had at least two films with $d_F < d_c$, so Kogan theory is not tenable.