Record-scale entanglement of quantum fields for measurement-based quantum computing

Olivier Pfister
University of Virginia
Students/Postdocs

Ph.D. Students

Sheng Feng (2005)  Physics faculty, Huazhong University, Wuhan
Raphael Pooser (2007)  Research staff, Oak Ridge National Laboratory
Daruo Xie (2007)  Finance faculty, Australian National University
Russell Bloomer (2010)  Staff, National Ground Intelligence Center
Matthew Pysher (2011)  Staff, ESI, Inc., now OC Tanner, Inc.
Reihaneh Shahrokhhshahi (2013)  Postdoc, SUNY Stony Brook
Moran Chen (2014)  Research scientist, Fibertek, Inc.
Pei Wang (2016)

Postdocs

Jietai Jing (2004-06)  Physics faculty, East China Normal University, Shanghai
Aye Win (2016-)

SSESAPS 2016
Past and present collaborators

Nicolas Menicucci
RMIT University, Melbourne

Steven Flammia
University of Sydney

Saewoo Nam
NIST

Aaron Miller
Albion College, now Quantum Opus, L.L.C.

Ady Arie
Tel-Aviv University

Murray Olsen
University of Queensland

Ashton Bradley
University of Otago

Peter Drummond
Swinburne University

Philip Battle
AdvR, Inc.

Joe Campbell
University of Virginia

Claude Fabre
UPMC, LKB, Paris

Nicolas Treps
UPMC, LKB, Paris

Paulo Nussenzveig
USP, São Paulo

Marcelo Martinelli
USP, São Paulo
Two flavors of QC
Two flavors of QC

1. Circuit-based QC

- Start in a separable state
- Quantum gates generate entanglement as quantum algorithm unfolds
One-way quantum computing: measurement-based

Uses cluster entangled states [Briegel & Raussendorf, PRL (2001)]

\[
\left( |0\rangle + |1\rangle \right) \left( |0\rangle + |1\rangle \right) \\
\downarrow \\
|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle \\
= |0\rangle|+\rangle + |1\rangle|-\rangle \\
= |+\rangle|0\rangle + |-\rangle|1\rangle
\]
One-way quantum computing: measurement-based

Uses cluster entangled states [Briegel & Raussendorf, PRL (2001)]

\[
\begin{align*}
&\left( |0\rangle + |1\rangle \right) \left( |0\rangle + |1\rangle \right) \\
\downarrow \\
&|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle \\
&= |0\rangle|+\rangle + |1\rangle|-\rangle \\
&= |+\rangle|0\rangle + |-\rangle|1\rangle
\end{align*}
\]
One-way quantum computing: measurement-based

Uses cluster entangled states [Briegel & Raussendorf, PRL (2001)]

\[
(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)
\]
\[
\downarrow
\]
\[
|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle
\]
\[
= |0\rangle|+\rangle + |1\rangle|_\rangle
\]
\[
= |+_\rangle|0\rangle + |-_\rangle|1\rangle
\]
One-way quantum computing: measurement-based

Uses cluster entangled states [Briegel & Raussendorf, PRL (2001)]

\[(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)\]
\[
\downarrow
\]
\[|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle - |1\rangle|1\rangle\]
\[= |0\rangle|+\rangle + |1\rangle|-\rangle\]
\[= |+\rangle|0\rangle + |-\rangle|1\rangle\]
One-way quantum computing: measurement-based

**Uses cluster entangled states** [Briegel & Raussendorf, PRL (2001)]

\[
\begin{align*}
(\ket{0} + \ket{1})(\ket{0} + \ket{1}) \\
\downarrow \\
\ket{0}\ket{0} + \ket{0}\ket{1} + \ket{1}\ket{0} - \ket{1}\ket{1} \\
= \ket{0}\ket{+} + \ket{1}\ket{-} \\
= \ket{+}\ket{0} + \ket{-}\ket{1}
\end{align*}
\]

\[2^N \text{ components}\]

\[N \text{ entanglement witnesses } X_i \bigotimes Z_j \quad j \in \mathcal{N}(i) \text{ (stabilizer group generators)}\]
Can also shape clusters with measurements

\( |+\rangle \quad |+\rangle \quad |+\rangle \)

CZ  CZ
Can also shape clusters with measurements

\[ |+\rangle |+\rangle |+\rangle \]

Measure Z (qubits)
i.e. Q (qumodes)
Can also shape clusters with measurements

\[
|+\rangle \quad |+\rangle \quad |+\rangle
\]

CZ  CZ

feed forward

Measure Z (qubits)
i.e. Q (qumodes)
Can also shape clusters with measurements

Measure $Z$ (qubits)
i.e. $Q$ (qumodes)
Can also shape clusters with measurements

feed forward

Measure $Z$ (qubits)
i.e. $Q$ (qumodes)
Can also shape clusters with measurements

Measure $Z$ (qubits)
i.e. $Q$ (qumodes)

Measure $X$ (qubits)
i.e. $P$ (qumodes)
Can also shape clusters with measurements

Measure Z (qubits)
\text{i.e. } Q \text{ (qumodes)}

Measure X (qubits)
\text{i.e. } P \text{ (qumodes)}
Can also shape clusters with measurements

Measure $Z$ (qubits)

Measure $X$ (qubits)

i.e. $Q$ (qumodes)

i.e. $P$ (qumodes)
Can also shape clusters with measurements

Measure $Z$ (qubits)

i.e. $Q$ (qumodes)

Measure $X$ (qubits)

i.e. $P$ (qumodes)

feed forward
Can also shape clusters with measurements

\[ |+\rangle \quad |+\rangle \quad |+\rangle \]

CZ   CZ

\[
\begin{array}{c}
\text{feed forward} \\
\text{Measure } Z \text{ (qubits)} \\
i.e. Q \text{ (qumodes)}
\end{array}
\]

\[
\begin{array}{c}
\text{feed forward} \\
\text{Measure } X \text{ (qubits)} \\
i.e. P \text{ (qumodes)}
\end{array}
\]
Can also shape clusters with measurements

Measure Z (qubits) i.e. Q (qumodes)
feed forward

Measure X (qubits) i.e. P (qumodes)
Can also shape clusters with measurements

Measure $Z$ (qubits)
i.e. $Q$ (qumodes)

Measure $X$ (qubits)
i.e. $P$ (qumodes)
Discrete vs. Continuous qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

**Qubits**

**Qumodes**

**Pauli group**

\[ G = \langle Z, X \rangle \]
Discrete vs. Continuous
qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

Pauli group

\[ G = \langle Z, X \rangle \]

Qumodes

Weyl-Heisenberg group

\[ G_\infty = \langle Z(\varpi), X(\xi) \rangle_{\varpi, \xi} \]
\[ \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi, \xi} \]

\[ Q = \frac{1}{\sqrt{2}} (a + a^\dagger) \]
\[ P = \frac{i}{\sqrt{2}} (a^\dagger - a) \]
\[ [Q, P] = i \]
\[ \Delta Q \Delta P = \frac{1}{2} \]
Discrete vs. Continuous qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

Pauli group
$G = \langle Z, X \rangle$

Weyl-Heisenberg group
$G_\infty = \langle Z(\omega), X(\xi) \rangle_{\omega, \xi}$
\[ \equiv \langle e^{i\omega Q}, e^{-i\xi P} \rangle_{\omega, \xi} \]

Qumodes

Computational basis

\[
Q = \frac{1}{\sqrt{2}}(a + a^\dagger) \\
\]
\[
P = \frac{i}{\sqrt{2}}(a^\dagger - a) \\
\]
\[
[Q, P] = i \\
\]
\[
\Delta Q \Delta P = \frac{1}{2} \\
\]
Discrete vs. Continuous
qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002);
Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

\( G = \langle Z, X \rangle \)

Pauli group

Qumodes

Weyl-Heisenberg group

\( G_\infty = \langle Z(\varpi), X(\xi) \rangle_{\varpi,\xi} \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi,\xi} \)

Computational basis

\( X \ | 0 \rangle = | 1 \rangle \)

\[
Q = \frac{1}{\sqrt{2}} (a + a^\dagger)
\]

\[
P = \frac{i}{\sqrt{2}} (a^\dagger - a)
\]

\[
[Q, P] = i
\]

\[
\Delta Q \Delta P = \frac{1}{2}
\]
Discrete vs. Continuous
qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

**Pauli** group

\[ G = \langle Z, X \rangle \]

**Weyl-Heisenberg** group

\[ G_{\infty} = \langle Z(\varpi), X(\xi) \rangle_{\varpi, \xi} \]

\[ \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi, \xi} \]

Computational basis

\[ X \ket{0} = \ket{1} \]

\[ X(\xi) \ket{q} = \ket{q + \xi} \]
Discrete vs. Continuous qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

Pauli group

\[ G = \langle Z, X \rangle \]

Computational basis

\[ X | 0 \rangle = | 1 \rangle \]

\[ Z | 1 \rangle = - | 1 \rangle \]

Qumodes

Weyl-Heisenberg group

\[ G_\infty = \langle Z(\varpi), X(\xi) \rangle_{\varpi, \xi} \]

\[ \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi, \xi} \]

\[ \Delta Q \Delta P = \frac{1}{2} \]

\[ Q = \frac{1}{\sqrt{2}}(a + a^\dagger) \]

\[ P = \frac{i}{\sqrt{2}}(a^\dagger - a) \]

\[ [Q, P] = i \]
Discrete vs. Continuous
qubits vs. qu modes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

**Qubits**

**Pauli** group

\( G = \langle Z, X \rangle \)

\[ X |0\rangle = |1\rangle \]

\[ Z |1\rangle = -|1\rangle \]

**Weyl-Heisenberg** group

\( G_\infty = \langle Z(\varpi), X(\xi) \rangle_{\varpi,\xi} \)

\[ \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi,\xi} \]

**Computational basis**

\[ X(\xi) |q\rangle = |q + \xi\rangle \]

\[ Z(\varpi) |q\rangle = e^{i\varpi q} |q\rangle \]

\[ Q = \frac{1}{\sqrt{2}} (a + a^\dagger) \]

\[ P = \frac{i}{\sqrt{2}} (a^\dagger - a) \]

\[ [Q, P] = i \]

\[ \Delta Q \Delta P = \frac{1}{2} \]
Discrete vs. Continuous
qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002);
Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

Pauli group
\[ G = \langle Z, X \rangle \]

Weyl-Heisenberg group
\[ G_\infty = \langle Z(\varpi), X(\xi) \rangle_{\varpi,\xi} \]
\[ \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi,\xi} \]

Computational basis
\[ X \mid 0 \rangle = \mid 1 \rangle \]
\[ Z \mid 1 \rangle = - \mid 1 \rangle \]

Hadamard basis

\[ X(\xi) \mid q \rangle = \mid q + \xi \rangle \]
\[ Z(\varpi) \mid q \rangle = e^{i\varpi q} \mid q \rangle \]

Qumodes

\[ Q = \frac{1}{\sqrt{2}} (a + a^\dagger) \]
\[ P = \frac{i}{\sqrt{2}} (a^\dagger - a) \]
\[ [Q, P] = i \]
\[ \Delta Q \Delta P = \frac{1}{2} \]
Discrete vs. Continuous
qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

**Qubits**

**Pauli group**
\[ G = \langle Z, X \rangle \]

**Computational basis**
\[ X |0\rangle = |1\rangle \]
\[ Z |1\rangle = -|1\rangle \]

**Hadamard basis**
\[ X |\pm\rangle = \pm |\pm\rangle \]

**Qumodes**

**Weyl-Heisenberg group**
\[ G_\infty = \langle Z(\varpi), X(\xi) \rangle_{\varpi, \xi} \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi, \xi} \]

**Computational basis**
\[ X(\xi) |q\rangle = |q + \xi\rangle \]
\[ Z(\varpi) |q\rangle = e^{i\varpi q} |q\rangle \]

\[ Q = \frac{1}{\sqrt{2}} (a + a^\dagger) \]
\[ P = \frac{i}{\sqrt{2}} (a^\dagger - a) \]
\[ [Q, P] = i \]
\[ \Delta Q\Delta P = \frac{1}{2} \]
Discrete vs. Continuous
qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

Pauli group
\[ G = \{ Z, X \} \]

Computational basis
\[ X |0\rangle = |1\rangle \]
\[ Z |1\rangle = -|1\rangle \]

Hadamard basis
\[ X |\pm\rangle = \pm |\pm\rangle \]

Qumodes

Weyl-Heisenberg group
\[ G_\infty = \{ Z(\varpi), X(\xi) \} \]
\[ \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle \]

\[ [Q, P] = i \]
\[ \Delta Q \Delta P = \frac{1}{2} \]

Fourier basis
\[ X(\xi) |q\rangle = |q + \xi\rangle \]
\[ Z(\varpi) |q\rangle = e^{i\varpi q} |q\rangle \]
Discrete vs. Continuous
qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002);
Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

Pauli group
\[ G = \langle Z, X \rangle \]

Computational basis
\[ X |0\rangle = |1\rangle \]
\[ Z |1\rangle = - |1\rangle \]

Hadamard basis
\[ X |\pm\rangle = \pm |\pm\rangle \]

Qumodes

Weyl-Heisenberg group
\[ G_\infty = \langle Z(\varpi), X(\xi) \rangle_{\varpi, \xi} \]
\[ \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi, \xi} \]

Fourier basis
\[ X(\xi) |p\rangle = e^{-i\xi p} |p\rangle \]

Hadamard basis
\[ X(\xi) |q\rangle = |q + \xi\rangle \]

\[ Q = \frac{1}{\sqrt{2}} (a + a^\dagger) \]
\[ P = \frac{i}{\sqrt{2}} (a^\dagger - a) \]
\[ [Q, P] = i \]
\[ \Delta Q \Delta P = \frac{1}{2} \]
Discrete vs. Continuous
qubits vs. qumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

Pauli group
\[ G = \langle Z, X \rangle \]

- \( Z \ket{1} = - \ket{1} \)
- \( X \ket{0} = \ket{1} \)

Hadamard basis
- \( X \ket{\pm} = \pm \ket{\pm} \)
- \( Z \ket{\pm} = \ket{\mp} \)

Qumodes

Weyl-Heisenberg group
\[ G_\infty = \langle Z(\varpi), X(\xi) \rangle_{\varpi, \xi} \]
\[ \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi, \xi} \]

- \( Z(\varpi) \ket{q} = e^{i\varpi q} \ket{q} \)
- \( X(\xi) \ket{q} = \ket{q + \xi} \)

Fourier basis
\[ X(\xi) \ket{p} = e^{-i\xi p} \ket{p} \]
Discrete vs. Continuous qubits vs. quumodes

Lloyd & Braunstein, PRL 82, 1784 (1999); Bartlett, Sanders, Braunstein, Nemoto, PRL 88 097904 (2002); Menicucci et al., PRL 97 110501 (2006); Gu et al., PRA 79 062318 (2009)

Qubits

Pauli group

\[ G = \langle Z, X \rangle \]

Computational basis

\[
X |0\rangle = |1\rangle \\
Z |1\rangle = -|1\rangle \\

Hadamard basis

\[
X |\pm\rangle = \pm |\pm\rangle \\
Z |\pm\rangle = |\mp\rangle \\

Qumodes

Weyl-Heisenberg group

\[ G_\infty = \langle Z(\varpi), X(\xi) \rangle_{\varpi, \xi} \]

\[ \equiv \langle e^{i\varpi Q}, e^{-i\xi P} \rangle_{\varpi, \xi} \]

\[ [Q, P] = i \]

\[ \Delta Q \Delta P = \frac{1}{2} \]

\[
X(\xi) |q\rangle = |q + \xi\rangle \\
Z(\varpi) |q\rangle = e^{i\varpi q} |q\rangle \\

Fourier basis

\[
X(\xi) |p\rangle = e^{-i\xi p} |p\rangle \\
Z(\varpi) |p\rangle = |p + \varpi\rangle \\

\[ Q = \frac{1}{\sqrt{2}} (a + a^\dagger) \]

\[ P = \frac{i}{\sqrt{2}} (a^\dagger - a) \]
Discrete vs. Continuous qubits vs. quumodes

**Qubits**

*Clifford* group

\[ N(G) = \langle H, P_{\frac{\pi}{2}}, C_X \rangle \]

**Quumodes**

\[ Q = \frac{1}{\sqrt{2}}(a + a^\dagger) \]

\[ P = \frac{i}{\sqrt{2}}(a^\dagger - a) \]

\[ [Q, P] = i \]

\[ \Delta Q \Delta P = \frac{1}{2} \]
Discrete vs. Continuous qubits vs. quumodes

**Qubits**

*Clifford* group

\[ \mathcal{N}(G) = \langle H, P_{\frac{\pi}{2}}, C_X \rangle \]

**Quumodes**

*Gaussian* operations (preserve Gaussian nature of Wigner function)

\[
\begin{align*}
Q &= \frac{1}{\sqrt{2}} (a + a^\dagger) \\
P &= \frac{i}{\sqrt{2}} (a^\dagger - a) \\
[Q, P] &= i \\
\Delta Q \Delta P &= \frac{1}{2}
\end{align*}
\]
Discrete vs. Continuous
qubits vs. qumodes

Qubits

**Clifford** group

\[ \mathcal{N}(G) = \langle H, P_\pi / 2, C_X \rangle \]

Qumodes

**Gaussian** operations (preserve Gaussian
nature of Wigner function)

\[ \mathcal{N}(G_\infty) : \text{at-most quadratic Hamiltonians} \]

- phase shifter
  \[ e^{-i\phi(a^\dagger a)} \]
- beam splitter
  \[ e^{-i\phi(a_1^\dagger a_2 + a_1 a_2^\dagger)} \]
- squeezers
  \[ e^{\kappa / 2 (a_1^\dagger a_2^\dagger - a_1 a_2)}, e^{\kappa (a_1^\dagger a_2^\dagger - a_1 a_2)} \]
- C-gates (QND)

\[ Q = \frac{1}{\sqrt{2}} (a + a^\dagger) \]
\[ P = \frac{i}{\sqrt{2}} (a^\dagger - a) \]
\[ [Q, P] = i \]
\[ \Delta Q \Delta P = \frac{1}{2} \]
Discrete vs. Continuous qubits vs. qumodes

Qubits

**Clifford** group

$$\mathcal{N}(G) = \langle H, P_{\frac{\pi}{2}}, C_X \rangle$$

$$C_X | 11 \rangle = | 10 \rangle$$

Qumodes

**Gaussian** operations (preserve Gaussian nature of Wigner function)

$$\mathcal{N}(G_{\infty})$$: at-most quadratic Hamiltonians

- phase shifter  $$e^{-i\phi(a^\dagger a)}$$
- beam splitter  $$e^{-i\phi(a_1^\dagger a_2 + a_1 a_2^\dagger)}$$
- squeezers  $$e^{\frac{\kappa}{2}(a^\dagger a^2 - a^2)}$$,  $$e^{\kappa(a_1^\dagger a_2^\dagger - a_1 a_2)}$$
- C-gates (QND)  $$e^{-iQ_1 P_2} | q_1, q_2 \rangle = | q_1, q_2 + q_1 \rangle$$

$$Q = \frac{1}{\sqrt{2}}(a + a^\dagger)$$

$$P = \frac{i}{\sqrt{2}}(a^\dagger - a)$$

$$[Q, P] = i$$

$$\Delta Q \Delta P = \frac{1}{2}$$
Discrete vs. Continuous qubits vs. qumodes

**Qubits**

*Clifford* group

\[ \mathcal{N}(G) = \langle H, P_{\frac{\pi}{2}}, C_X \rangle \]

- \[ C_X \ket{11} = \ket{10} \]
- \[ C_Z \ket{11} = -\ket{11} \]

**Qumodes**

*Gaussian* operations (preserve Gaussian nature of Wigner function)

\[ \mathcal{N}(G_\infty) : \text{at-most quadratic Hamiltonians} \]

- phase shifter \[ e^{-i\phi(a^\dagger a)} \]
- beam splitter \[ e^{-i\phi(a_1^\dagger a_2 + a_1 a_2^\dagger)} \]
- squeezers \[ e^{\frac{\kappa}{2}(a_1^\dagger a_1^\dagger - a_1 a_1)} e^{\kappa (a_1^\dagger a_2^\dagger - a_1 a_2)} \]
- C-gates (QND) \[ e^{-iQ_1 P_2} \ket{q_1, q_2} = \ket{q_1, q_2 + q_1} \]
\[ e^{-iQ_1 Q_2} \ket{q_1, q_2} = e^{-iq_1 q_2} \ket{q_1, q_2} \]

\[ Q = \frac{1}{\sqrt{2}}(a + a^\dagger) \]
\[ P = \frac{1}{\sqrt{2}}(a^\dagger - a) \]
\[ [Q, P] = i \]
\[ \Delta Q \Delta P = \frac{1}{2} \]
Quantum Optics

\[ \vec{E}(\vec{r}, t) = \sqrt{\frac{\hbar \omega}{2\varepsilon_0 V}} \hat{e} \left[ Q \cos(\vec{k} \cdot \vec{r} - \omega t) + P \sin(\vec{k} \cdot \vec{r} - \omega t) \right] \]

\[ Q = a + a^\dagger \quad P = (a - a^\dagger)/i \quad N = a^\dagger a \]

\[ [Q, P] = 2i \quad \Delta Q\Delta P \geq 1 \]
Quantum Optics

\[ E(\vec{r}, t) = \sqrt{\frac{\hbar \omega}{2 \varepsilon_0 V}} \hat{e} \left[ Q \cos(\vec{k} \cdot \vec{r} - \omega t) + P \sin(\vec{k} \cdot \vec{r} - \omega t) \right] \]

\[ Q = a + a^\dagger \quad P = (a - a^\dagger)/i \quad N = a^\dagger a \]

\[ [Q, P] = 2i \quad \Delta Q \Delta P \geq 1 \]
Quantum Optics

\[ \vec{E}(\vec{r}, t) = \sqrt{\frac{\hbar \omega}{2\varepsilon_o V}} \hat{e} \left[ Q \cos(\vec{k} \cdot \vec{r} - \omega t) + P \sin(\vec{k} \cdot \vec{r} - \omega t) \right] \]

\[ Q = a + a^\dagger \quad P = (a - a^\dagger)/i \quad N = a^\dagger a \]

\[ [Q, P] = 2i \quad \Delta Q \Delta P \geq 1 \]

EPR state: eigenstate of \( Q_1 - Q_2 \) and \( P_1 + P_2 \) (unnormalized, singular)

\[ \int_{-\infty}^{+\infty} |q\rangle_1 |q\rangle_2 dq = \int_{-\infty}^{+\infty} |p\rangle_1 - |p\rangle_2 dp = \sum_{n=0}^{\infty} |n\rangle_1 |n\rangle_2 \]

Einstein, Podolsky, and Rosen, PR 47, 777 (1935)
van Enk, PRA 60, 5095 (1999)
(Schmidt basis)
Quantum Optics

\[ \vec{E}(\vec{r}, t) = \sqrt{\frac{\hbar \omega}{2\varepsilon_0 V}} \hat{e} \left[ Q \cos(\vec{k} \cdot \vec{r} - \omega t) + P \sin(\vec{k} \cdot \vec{r} - \omega t) \right] \]

\[ Q = a + a^\dagger \quad P = \frac{(a - a^\dagger)}{i} \quad N = a^\dagger a \]

\[ [Q, P] = 2i \quad \Delta Q \Delta P \geq 1 \]

EPR state: eigenstate of \( Q_1 - Q_2 \) and \( P_1 + P_2 \) (unnormalized, singular)

\[
\int_{-\infty}^{+\infty} |q\rangle_1 |q\rangle_2 dq = \int_{-\infty}^{+\infty} |p\rangle_1 - p\rangle_2 dp = \sum_{n=0}^{\infty} |n\rangle_1 |n\rangle_2 \\
\]

Einstein, Podolsky, and Rosen, PR 47, 777 (1935) \quad van Enk, PRA 60, 5095 (1999)

(Schmidt basis)

Two-mode squeezed state of \( Q_1 - Q_2 \) & \( P_1 + P_2 \) (normalized, smooth)

\[
|\psi(r)\rangle_{12} = \frac{1}{\cosh^2 r} \sum_{n=0}^{\infty} \tanh^n r |n\rangle_1 |n\rangle_2 \xrightarrow{r \to \infty} |EPR\rangle_{12} \\
\]

Einstein, Podolsky, and Rosen, PR 47, 777 (1935) \quad van Enk, PRA 60, 5095 (1999)

(Schmidt basis)
Quantum Optics

\[ \bar{E}(\vec{r}, t) = \sqrt{\frac{\hbar \omega}{2\varepsilon_0 V}} \hat{e} \left[ Q \cos(\vec{k} \cdot \vec{r} - \omega t) + P \sin(\vec{k} \cdot \vec{r} - \omega t) \right] \]

\[ Q = a + a^\dagger \quad P = (a - a^\dagger) / i \quad N = a^\dagger a \]

\[ [Q, P] = 2i \quad \Delta Q \Delta P \geq 1 \]

EPR state: eigenstate of \( Q_1 - Q_2 \) and \( P_1 + P_2 \) (unnormalized, singular)

\[ \int_{-\infty}^{+\infty} \ket{q}_1 \ket{q}_2 \, dq = \int_{-\infty}^{+\infty} \ket{p}_1 - \ket{p}_2 \, dp = \sum_{n=0}^{\infty} \ket{n}_1 \ket{n}_2 \]

Einstein, Podolsky, and Rosen, PR 47, 777 (1935)

van Enk, PRA 60, 5095 (1999)

(Schmidt basis)

"Variance-based entanglement witnesses", or "nullifiers"

Two-mode squeezed state of \( Q_1 - Q_2 \) & \( P_1 + P_2 \) (normalized, smooth)

\[ \ket{\psi(r)}_{12} = \frac{1}{\cosh^2 r} \sum_{n=0}^{\infty} \tanh^n r \ket{n}_1 \ket{n}_2 \quad r \to \infty \quad \ket{EPR}_{12} \]
Qubits to Qudits to Qumodes
Qubits to Qudits to Qumodes

Qubits

\[ |0\rangle_a |0\rangle_b + |1\rangle_a |1\rangle_b \]
Qubits to Qudits to Qumodes

**Qubits**

\[ |0\rangle_a |0\rangle_b + |1\rangle_a |1\rangle_b \]

**Qudits**

\[
\sum_{n=0}^{\infty} |n\rangle_a |n\rangle_b
\]
Qubits to Qudits to Qumodes

Qubits

$$|0\rangle_a |0\rangle_b + |1\rangle_a |1\rangle_b$$

Qudits

$$\sum_{n=0}^{\infty} |n\rangle_a |n\rangle_b$$

Qumodes

$$\int_{-\infty}^{\infty} |q\rangle_a |q\rangle_b \, dq$$
Qubits to Qudits to Qumodes

**Qubits**

\[ |0\rangle_a |0\rangle_b + |1\rangle_a |1\rangle_b \]

**Qudits**

\[ \sum_{n=0}^{\infty} |n\rangle_a |n\rangle_b \]

**Qumodes**

\[ \int_{-\infty}^{\infty} |q\rangle_a |q\rangle_b \, dq = \int_{-\infty}^{\infty} |p\rangle_a |p\rangle_b \, dp \]
Qubits to Qudits to Qumodes

**Qubits**

\[ |0\rangle_a |0\rangle_b + |1\rangle_a |1\rangle_b \]

**Qudits**

\[
\sum_{n=0}^{\infty} |n\rangle_a |n\rangle_b
\]

**Qumodes**

\[
\int_{-\infty}^{\infty} |q\rangle_a |q\rangle_b \, dq = \int_{-\infty}^{\infty} |p\rangle_a - p\rangle_b \, dp
\]

SAME: Schmidt decomposition of an EPR state

[S. van Enk, PRA 1999]
Qubits to Qudits to Qumodes

Qubits

$$|0\rangle_a |0\rangle_b + |1\rangle_a |1\rangle_b$$

Qudits

$$\sum_{n=0}^{\infty} |n\rangle_a |n\rangle_b$$

Qumodes

$$\int_{-\infty}^{\infty} |q\rangle_a |q\rangle_b dq = \int_{-\infty}^{\infty} |p\rangle_a |-p\rangle_b dp$$

SAME:

Schmidt decomposition of an EPR state

[S. van Enk, PRA 1999]

No such clear connection for multipartite entanglement
A lesson in research from Ted Hänsch...

goal-oriented

curiosity-driven
The eigenmodes of a cavity form a large ensemble of classically coherent modes.

**Linear gain**

Laser

...  

...  

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)  
(10^6 Cmodes oscillating in phase)

**Nonlinear gain**

Optical Parametric Oscillator (OPO)

...why not turn it into a quantum computer?
The eigenmodes of a cavity form a large ensemble of classically coherent modes.

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)
(10⁶ C-modes oscillating in phase)

...why not turn it into a quantum computer?
The eigenmodes of a cavity form a large ensemble of classically coherent modes.

**Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)**

(10^6 Cmodes oscillating in phase)

**...why not turn it into a quantum computer?**
The eigenmodes of a cavity form a large ensemble of classically coherent modes. Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) (10^6 Cmodes oscillating in phase).

...why not turn it into a quantum computer?

Optical Parametric Oscillator (OPO)
The eigenmodes of a cavity form a large ensemble of classically coherent modes.

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) (10^6 Cmodes oscillating in phase)

...why not turn it into a quantum computer?

\[ \omega_p = \omega_m + \omega_n \]
The optical cavity: a naturally large set of oscillators...

The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) 
(10^6 Cmodes oscillating in phase)

...why not turn it into a quantum computer?
The optical cavity: a naturally large set of oscillators...

The eigenmodes of a cavity form a large ensemble of classically coherent modes.

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) 
(10^6 Cmodes oscillating in phase)

...why not turn it into a quantum computer?

Optical Parametric Oscillator (OPO)

linear gain

nonlinear gain

quantum OFC
The optical cavity: a naturally large set of oscillators...

The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC)
(10^6 Cmodes oscillating in phase)

...why not turn it into a quantum computer?
The optical cavity: a naturally large set of oscillators...

The eigenmodes of a cavity form a large ensemble of classically coherent modes

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) (10^6 Cmodes oscillating in phase)

...why not turn it into a quantum computer?
The optical cavity: a naturally large set of oscillators...

The eigenmodes of a cavity form a large ensemble of classically coherent modes.

Carrier-envelope-phase locked mode-locked laser = optical frequency comb (OFC) 
$(10^6 \text{ Cmodes oscillating in phase})$

...why not turn it into a quantum computer?

Multiparticle cluster entanglement in one fell swoop:
a top-down, large-scale quantum register.
One-Way Quantum Computing in the Optical Frequency Comb

Nicolas C. Menicucci, Steven T. Flammia, and Olivier Pfister
Parallel Generation of Quadripartite Cluster Entanglement in the Optical Frequency Comb

Matthew Pysher,¹ Yoshichika Miwa,² Reihaneh Shahrokhshahi,¹ Russell Bloomer,¹ and Olivier Pfister¹,*

Entanglement gets scaled up in an optical frequency comb
Using a single nonlinear optical element, researchers have entangled dozens of the comb’s optical modes.

www.physicstoday.org  September 2011  Physics Today  21
Experimental Realization of Multipartite Entanglement of 60 Modes of a Quantum Optical Frequency Comb

Moran Chen,¹ Nicolas C. Menicucci,²,* and Olivier Pfister¹,†
¹Department of Physics, University of Virginia, Charlottesville, Virginia 22903, USA
²School of Physics, The University of Sydney, Sydney, New South Wales 2006, Australia
Akira Furusawa’s group: sequential entanglement of $10^4$ qumodes (2 at a time)
Akira Furusawa’s group: sequential entanglement of $10^4$ qumodes (2 at a time)
Entangling the QOFC by interfering \textit{frequency}-shifted EPR pairs
Entangling the QOFC by interfering \textit{frequency}-shifted EPR pairs
Entangling the QOFC by interfering *frequency*-shifted EPR pairs
Entangling the QOFC by interfering frequency-shifted EPR pairs
Entangling the QOFC by interfering \textit{frequency}-shifted EPR pairs
Entangling the QOFC by interfering *frequency*-shifted EPR pairs

\[ \omega_p = \omega_m + \omega_n \]

\[ p_Y = -1 \]

\[ p_Z = 1 \]

Dual-rail quantum wire
Experimental setup

- Laser1
- Laser2 (with flip mirror)
- Laser3

 PLL

- AOM
- EOM
- OPO
- PPKTP
- OPO comb
- PDH Lock

- HWP
- Filter
- Cavity
- LO
- PBS
- SA

2mΔω

Ω = (n + 1/2)Δω

Entanglement Verification

9.2 MHz

AOM

70 MHz

EOM

70 MHz

PDH Lock

zzz

PPP

YYY

PPKTP

zzz

YYYY

PDH Lock

9.2 MHz

Ψ = (n + 1/2)ΔΔ

Entanglement Verification

θ₀

ω_{off} ± 70 MHz
2-crystal, multipartite entangling OPO
One Belt Y 1.5 Y pump Z pump

Noise Level (dB)

Local Oscillator Phase (arb. unit)
One Belt Y pump Z pump

Noise Level (dB)

Local Oscillator Phase (arb. unit)

PZT turning point
One Belt Y Z

Noise Level (dB)

Local Oscillator Phase (arb. unit)

Y pump Z pump

PZT turning point
Noise Level (dB) vs Local Oscillator Phase (arb. unit) for PZT turning point.
Noise Level (dB)

Local Oscillator Phase (arb. unit)

PZT turning point
One Belt
Z 6.5
Y pump
Z pump

Noise Level (dB)

PZT turning point

Local Oscillator Phase (arb. unit)
One Belt

Y pump Z pump

Noise Level (dB)

Local Oscillator Phase (arb. unit)

PZT turning point
One Belt

Noise Level (dB)

Local Oscillator Phase (arb. unit)

PZT turning point
<table>
<thead>
<tr>
<th>Noise Level (dB)</th>
<th>Local Oscillator Phase (arb. unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

(a) y-pump-centered 0.5 FSR  
(b) y-pump-centered 1.5 FSR  
(c) y-pump-centered 2.5 FSR  
(d) y-pump-centered 3.5 FSR  
(e) y-pump-centered 4.5 FSR  
(f) y-pump-centered 5.5 FSR  
(g) y-pump-centered 6.5 FSR  
(h) y-pump-centered 7.5 FSR  
(i) y-pump-centered 8.5 FSR  
(j) y-pump-centered 9.5 FSR  
(k) z-pump-centered 10.5 FSR  
(l) z-pump-centered 11.5 FSR  
(m) z-pump-centered 12.5 FSR  
(n) z-pump-centered 13.5 FSR  
(o) z-pump-centered 14.5 FSR  
(p) z-pump-centered 15.5 FSR  
(q) z-pump-centered 16.5 FSR  
(r) z-pump-centered 17.5 FSR  
(s) z-pump-centered 18.5 FSR  
(t) z-pump-centered 19.5 FSR  
(u) z-pump-centered 20.5 FSR  
(v) z-pump-centered 21.5 FSR  
(w) z-pump-centered 22.5 FSR  
(x) z-pump-centered 23.5 FSR  
(y) z-pump-centered 24.5 FSR  
(z) z-pump-centered 25.5 FSR  

Local Oscillator Phase (arb. unit)
How many qumodes?
Phasematching bandwidth of our PPKTP crystal


\[ \nu_1 + \nu_2 = \frac{\nu_p}{2} \]

SHG peak
SFG “peak”
How many qumodes?
Phasematching bandwidth of our PPKTP crystal


\[ \frac{\nu_1 + \nu_2}{2} = \nu_p \]

- **SHG peak**
- **SFG “peak”**

- **3.2 THz**
- **6700 qumodes (w/o dispersion)**

\[ \nu_1 \]
\[ \nu_2 \]
Scalable, universal QC with a single OPO with hybrid time-frequency entanglement

One-way quantum computing with arbitrarily large time-frequency continuous-variable cluster states from a single optical parametric oscillator

Rafael N. Alexander,1,2 Pei Wang,3 Niranjan Sridhar,3 Moran Chen,3 Olivier Pfister,3,* and Nicolas C. Menicucci1,2,†
Scalable, universal QC with a single OPO with hybrid time-frequency entanglement.
Scalable, universal QC with a single OPO with hybrid time-frequency entanglement

Frequency-encoded quantum wires

Frequency-encoded quantum wires

Physical Review A 94, 032327 (2016)

One-way quantum computing with arbitrarily large time-frequency continuous-variable cluster states from a single optical parametric oscillator

Rafael N. Alexander,1,2 Pei Wang,3 Niranjan Sridhar,3 Moran Chen,3 Olivier Pfister,3,* and Nicolas C. Menicucci2,1,†
Scalable, universal QC with a single OPO with hybrid time-frequency entanglement

Frequency-encoded quantum wires

* PHYSICAL REVIEW A 94, 032327 (2016) *

One-way quantum computing with arbitrarily large time-frequency continuous-variable cluster states from a single optical parametric oscillator

Rafael N. Alexander,1,2 Pei Wang,3 Niranjan Sridhar,3 Moran Chen,3 Olivier Pfister,3,* and Nicolas C. Menicucci1,2,†
Scalable, universal QC with a single OPO with hybrid time-frequency entanglement

Frequency-encoded quantum wires

OPO

LO

HWP

PBS

Y-polarized even frequencies

Z-polarized even frequencies

Odd frequencies

(2p + 1)Δν

(2p - 1)Δν

(Δν, δν)

Frequency-encoded quantum wires

PHYSICAL REVIEW A 94, 032327 (2016)

One-way quantum computing with arbitrarily large time-frequency continuous-variable cluster states from a single optical parametric oscillator

Rafael N. Alexander,1,2 Pei Wang,3 Niranjan Sridhar,3 Moran Chen,3 Olivier Pfister,3,* and Nicolas C. Menicucci1,2,†
This is a square-grid lattice
This is a square-grid lattice
Conclusion

We have entangled 60 qumodes of the quantum optical frequency comb into one and two dual-rail quantum wires

- This is the largest simultaneously entangled cluster state to date. (We have reason to believe ~ 3000 modes are entangled in our case)
  - Furusawa (Tokyo) demonstrated a one-million-mode cluster state, 2 at a time
  - Treps (UPMC) demonstrated advanced quantum processing over ~10 modes

- Quantum wires from D OPOs can be used to create D-hypercubic lattices

- Scalable, universal quantum computer with a single OPO proposed (quantum error correction not included)

- There exists a fault tolerance threshold for CVQC

Menicucci PRL 2014